

Scaling laws for free piston Stirling engine design: Benefits and challenges of miniaturization



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ABSTRACT

This work explores the scaling effects for FPSE (free piston Stirling engines), which are known for their simple architecture and potentially high thermodynamic performances. Scaling laws are given and their potential for miniaturization is highlighted. A simple model which allows the design of the geometrical parameters of the heat exchangers, the regenerator and the masses of the pistons is proposed. It is based on the definition of six characteristic dimensionless groups. They are derived from the physics underlying the behavior of the free piston Stirling machine and their relevancy is backed up by comparisons between documented Stirling engines from the literature. Keeping constant values for each group throughout the scaling range theoretically ensures constant performance. The main losses of Stirling engine (heat conduction loss, reheat loss in the regenerator, pressure drop and gas-spring hysteresis) can be expressed as a function of the geometrical and operating parameters. Additionally, the consequences of leakage due to the manufacturing precision of pistons architectures are underlined. From the proposed scaling laws, potential power and efficiency of Stirling cycle engines at a millimeter scale can be anticipated. It appears that the power density increases with miniaturization. It is also shown that the dynamic masses related to the engine size are increased when scaling down and that the gap leakage presents the highest detrimental effects on the efficiency. These results call for dedicated architectures for micro-engines.

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1. Introduction

The miniaturization of thermo-mechanical devices has become an increasing trend to match the portable power demand. With moderate efficiencies, micro-engines using hydrocarbon fuels combustion as a heat source make the realization of high density powering system possible [1–3]. They aim at delivering electrical power ranging from dozens of milliwatts to a few watts. These devices could be manufactured by MEMS (microelectromechanical systems) process approaches and then be ultimately integrated to global units of sensors, actuators and microprocessor as “power plants on a chip”.

The Stirling cycle is one of the most interesting choices because of external heat source and no valves. It offers different possible applications: engine, cooler or heat pump. The FPSE (free piston Stirling engine) architecture is favorable for miniaturization because of its simple mechanical arrangement and the lack of

complex kinematic linkage. Fig. 1a presents a generic scheme of a FPSE which underlines the basic components:

- Two heat exchangers (heater, cooler). They allow the gas to be kept at constant high T_h and low T_c temperatures.
- The regenerator. This component is usually made of a porous matrix. It periodically stores and releases heat from and to the gas, lowering the external heat needed to keep the hot gas temperature.
- Two pistons (classically denoted piston and displacer). They ensure the proper dynamics of the FPSE. The displacer transfers the gas to the cold or the hot side of the engine (see arrows in Fig. 1) whereas the mechanical work is extracted from the piston motion. Note that the displacer also ensures a thermal isolation between the hot and cold sides of the engine.

Based on a given Stirling engine, models have been proposed for optimization purposes [4–7]. The complexity is increased in the case of FPSE [8,9] because of the strong coupling between the dynamic and thermodynamics equilibriums. The effect of a drastic miniaturization on the performances cannot be easily performed

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Nomenclature

A_d	displacer area (m ²)
A_p	piston area (m ²)
A_{rod}	rod area (m ²)
A_{ff}	free flow area (m ²)
c_d	displacer mechanical dissipation (N m ⁻¹ s)
c_p	piston mechanical dissipation (N m ⁻¹ s)
d_w	wire diameter of the matrix regenerator (m)
D_d	displacer diameter (m)
f	frequency (Hz)
f_F	friction factor (–)
h	convection coefficient (W m ⁻² K ⁻¹)
k	thermal conductivity of gas (W m ⁻¹ K ⁻¹)
L_i	length of component exchanger (m)
L_{ref}	reference length ($L_{ref} = V_{sw}^{1/3}$) (m)
M	total mass of gas (kg)
m_d	mass of a displacer (kg)
m_p	mass of a piston (kg)
m_i	mass of a gas within component i (kg)
n_x	number of channels (–)
P_m	mechanical power (W)
p	pressure (N m ⁻²)
R	ideal mass gas constant (J kg ⁻¹ K ⁻¹)
r_{hi}	hydraulic radius of exchanger (m)
T_c	cold temperature (K)
T_h	hot temperature (K)
T_w	wall temperature (K)
u_i	mean gas velocity in component i (m s ⁻¹)

V_i	volume of component i (m ³)
V_{sw}	displacement (m ³)
V_{swe}	expansion space swept volume (m ³)
V_{swc}	compression space swept volume (m ³)

Greek symbols

α	swept volume phase angle (rad)
Δp	Pressure loss (N m ⁻²)
ε	scale (–)
η_r	regenerator efficiency (–)
κ	swept volume ratio (–)
μ	dynamic viscosity (N m ⁻² s)
ρ	material density (kg m ⁻³)
ω	pulsation (rad s ⁻¹)
σ_i	dimensionless mass in component i ($\sigma_i = m_i/M$)
$\dot{\sigma}_i$	Dimensionless mass flow ($\dot{\sigma}_i = \dot{m}_i/(\omega M)$)
Φ	porosity of the regenerator (–)

Subscripts

bp	piston gas spring space
bd	displacer gas spring space
c	cooler
d	displacer
h	heater
p	piston
r	regenerator
ref	reference
rod	Rod of displacer

using these models. Then, a dedicated approach is required to obtain design scaling rules and provide relevant information for novel architectures.

Only a few works dealt with the specific issue of the size effect on the performances of miniature Stirling machines. Nakajima [10]

performed a scale analysis on each of the elementary effects which govern the behavior of the Stirling engine but these effects were considered independently.

Many works focused on the study of the regenerator because of the drastic repercussion of its effectiveness on the efficiency of the

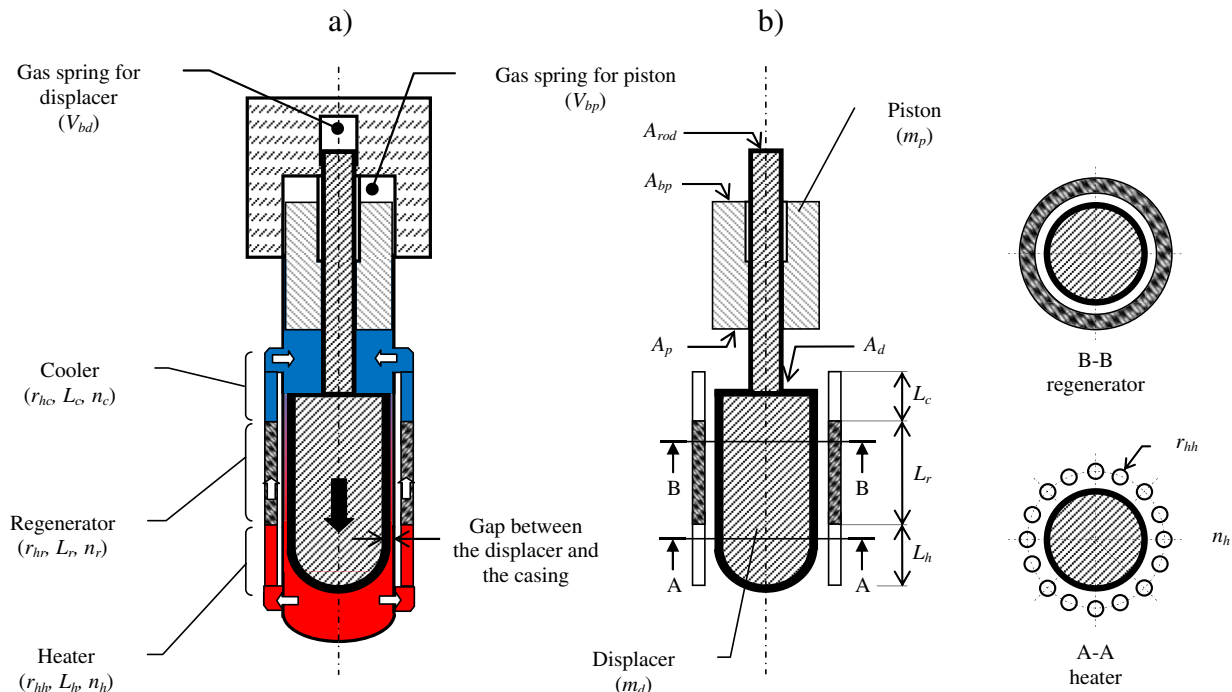


Fig. 1. a) Simplified geometry for the study, b) main parameters.

Stirling machine. For cryocoolers, Peterson [11] developed an approach of scaling effects on the regenerator effectiveness and the axial thermal conduction. Yang and Chung [12] extended the study of the regenerator to take into account the void volumes, as well as the frictional and the viscous losses. Conclusions suggest that millimeter size machines are viable providing that low conductivity material is used. Nonetheless, because the considered pressure and the frequency were kept constant in these studies, restrictive deductions may have been obtained. The operating parameters (pressure and frequency) are indeed likely to be modified during scaling.

Simple modeling approaches that can be performed without computational needs are called FOM (first-order methods) according to the Martini's nomenclature [13]. They represent effective and useful models for preliminary design. The simplest FOM gives a relationship between the engine output power the mean pressure, the operating frequency, the displacement and a dimensionless number: the Beale number BeN defined in Eq. (1) [14]:

$$BeN = P_m / (V_{sw} p_{mean} \omega) \quad (1)$$

The Beale number is almost constant (≈ 0.015) for Stirling engines operating at a temperature ratio $\tau = T_c/T_h$ of about 1/3 and for engines which present the same dead volume ratio χ and phase angle α [14].

This relation suggests that the power is related to parameters which can be modified to compensate the size effects. As an example, the decrease of the power when using smaller displacement (V_{sw}) can be compensated by an increased frequency or pressure. However, their effects for the engine are questionable: What are the effects of the frequency and pressure on the exchanger or the regenerator performances? Can these components be geometrically scaled while maintaining their performances? Therefore, the Beale number is incomplete and a coupled analysis which includes the engine thermal components is needed here.

Aiming at a more accurate prediction of the power, the Beale number has been extended to include the regenerator, the engine geometry and the working gas parameters [15,16]. These strategies stem on correlations defined from experimental operating measurements of Stirling engines whose displacements range from 8 dm³ to 20 dm³. The main sources of losses (friction, leakages) can then be related to the engine parameters (frequency, pressure, type of gas, relative dead volumes). These approaches are accurate as far as the derived scaled engines are close to the displacement range. The validity of the power prediction and correlation laws for significantly smaller volumes may be questionable. Moreover, these analyses were not performed for FPSE for which too few experimental data are available in the literature.

We propose to develop a coupled approach to obtain novel FOM allowing easy preliminary designs of FPSE. Each of the parameters defined in Fig. 1 will then be evaluated for a given displacement (i.e. a given scale) and a given pressure which is usually bounded by technological constraints.

The proposed FOM is based on the similitude theory applied to the relevant physics pertaining to FPSE: thermodynamics, conservation of mass, momentum and energy for the gas flow within the heat exchangers and the regenerator, mechanical dynamic equilibrium for the piston and the displacer. Six different types of DGs (dimensionless groups) are then obtained. Five DGs are common to all types of Stirling engines and one additional DG is proposed to account for the dynamic in the case of FPSE. The DGs are evaluated for a reference engine. Then, the parameters of the basic components of the machine (see Fig. 1), can be evaluated keeping constant values for the DGs.

From the derived geometry, the effect of scaling on the classical losses in Stirling engine (conduction, regenerator reheat losses and pressure drop and gas spring hysteresis) can be studied. The sealing of piston is underlined as a major source of losses [2]. In this study, this effect is taken into account, evaluating the gas leakage within gap between the displacer and the casing (see Fig. 1). Finally, the evolution of the power density, the efficiency and the major losses are studied. Finally recommendations regarding architecture and process are suggested for micro-Stirling engine design.

2. Similitude approach for the free piston Stirling engines

Previous works about the similitude theory applied to Stirling engines are based on the underlying physics of the Stirling engine [17,18]. The basic equations of thermodynamics, gas flow (conservation of mass, momentum and energy) are written using dimensionless variables. Dimensionless groups of parameters are then enhanced and chosen as representative. To take into account flow friction and thermal exchange, usual dimensionless groups are added. In addition, the hot and cold temperatures are assumed to remain constant during the scaling process and some assumptions on the kinematic and the geometry are made. This strategy is recalled here and the DGs are established in a comprehensive presentation.

The obtained DGs were limited to kinematic Stirling engine. In this work, the method is extended to the study of the mechanical aspects of FPSE and additional DGs are established.

The proposed dimensionless variables are defined as:

- Pressure: $p^* = \frac{p}{p_{mean}} = p \frac{V_{sw}}{MRT_{ref}}$
- Mass within component i : $\sigma_i = m_i/M$
- Mass flow rate through component i : $\dot{\sigma}_i = \dot{m}_i/(\omega M)$
- Temperature: $T^* = T/T_{ref}$
- Time: $t^* = \omega t$
- Length: $x^* = x/L_{ref}$

2.1. Kinematic and geometry

A kinematic and geometric similarity is assumed. The phase angle, the swept volume ratio of the piston and displacer are constant.

Because the ratios of the swept volume to dead volumes are supposed to remain fixed, the first DG is then defined as:

$$DG_{Vi} = \frac{r_{hi}^2 L_i n_i}{V_{sw}} \quad (2)$$

In which i stands for the cooler ($i = c$), the heater ($i = h$) or the regenerator ($i = r$) component. Note that only the meaningful terms are kept which is why the π value does not appear in the numerator of Eq. (2).

2.2. Thermodynamics

From the ideal isothermal model assuming perfect gas law and sinusoidal motions, the power output can be expressed. This model, used in numerous studies, is called the Schmidt analysis and allows analytical expressions for the engine [14].

The power can then be expressed as:

$$P_m = \omega p_{mean} V_{sw} \times \left(\frac{1}{2\pi} \frac{1}{2\beta} \frac{V_{swe}}{V_{sw}} \frac{\kappa(\tau-1)\sin(\alpha)}{\sqrt{\tau^2 + \kappa^2 + 2\kappa\tau\cos(\alpha)}} \left(1 - \sqrt{1 - \beta^2} \right) \right) \quad (3)$$

In which τ , κ and β are defined in Eq. (4):

$$\begin{aligned}\tau &= \frac{T_c}{T_h} \\ \kappa &= \frac{V_{swc}}{V_{swe}} \\ \beta &= \frac{\sqrt{2\tau\kappa \cos(\alpha) + \tau^2 + \kappa^2}}{2/V_{swe}(V_c + V_r T_c/T_r + V_h T) + \kappa + \tau}\end{aligned}\quad (4)$$

The assumption of kinematic and geometric similarity leads to constant κ , α and V_{swe}/V_{sw} . Consequently, the Beale number Be_N , given in Eq. (1), can be defined from Eq. (3). Be_N is then chosen as the second DG.

2.3. Gas flow equations

The energetic similarity is obtained from the 1D behavior of an ideal gas flow within the heat exchangers and the regenerator. The fundamental conservation equations (mass, momentum and energy) are given by the Eqs. (5)–(7) respectively:

$$\frac{\partial \rho_i}{\partial t} + \frac{\partial}{\partial x} \left(\frac{\dot{m}_i}{A_{ff}} \right) = 0 \quad (5)$$

$$\frac{\partial}{\partial t} \left(\frac{\dot{m}_i}{A_{ff}} \right) + \frac{\partial}{\partial x} \left(\frac{\dot{m}_i}{A_{ff}} u_i \right) = -\frac{\partial p}{\partial x} - \frac{1}{2r_{hi}} f_F \frac{\dot{m}_i |\dot{m}_i|}{\rho A_{ff}^2} \quad (6)$$

$$\begin{aligned}\frac{\partial}{\partial t} \rho_i \left(c_v T + \frac{1}{2} \frac{\dot{m}_i^2}{\rho_i^2 A_{ff}^2} \right) + \left(c_v T + \frac{p}{\rho_i} + \frac{1}{2} \frac{\dot{m}_i^2}{\rho_i^2 A_{ff}^2} \right) \frac{\partial}{\partial x} \left(\frac{\dot{m}_i}{A_{ff}} u_i \right) \\ = h \frac{4\pi r_{hi}}{A_{ff}} (T_w - T)\end{aligned}\quad (7)$$

In which the free flow area is such as $A_{ff} = n_i \pi r_{hi}^2$. For the momentum Eq. (6), the fluid dissipation is expressed using the Fanning friction factor f_F .

Using the dimensionless variables and noting that for a considered 1D slug of fluid, one can write: $\rho_i = p/RT_i = (1/A_{ff})(M/L_{ref})(\partial \sigma_i / \partial x^*)$ and $\rho_i u_i = (\omega M/A_{ff}) \dot{\sigma}_i$ Eqs. (5)–(7) are written again as:

$$\frac{\partial}{\partial t^*} \left(\frac{\partial \sigma_i}{\partial x^*} \right) + \frac{\partial \dot{\sigma}_i}{\partial x^*} = 0 \quad (8)$$

$$\frac{\partial \dot{\sigma}_i}{\partial t^*} + \frac{\partial \dot{\sigma}_i u_i}{\partial x^*} = -\frac{1}{\gamma} \left(\frac{A_{ff} \sqrt{\gamma R T_{ref}}}{\omega L_{ref}^3} \right)^2 \frac{L_{ref}^2}{A_{ff}} \frac{\partial p^*}{\partial x^*} - f_F \frac{L_{ref}}{2r_{hi}} \frac{L_{ref}^2}{A_{ff}} \dot{\sigma}_i |\dot{\sigma}_i| \quad (9)$$

$$\begin{aligned}\frac{\partial}{\partial t^*} \left(\frac{\partial \sigma_i}{\partial x^*} \left(\frac{1}{\gamma} T^* + \frac{1}{2} \left(\frac{\omega L_{ref}^3}{A_{ff} \sqrt{\gamma R T_{ref}}} \right)^2 (\gamma - 1) \left(\frac{T^* \dot{\sigma}_i}{p^*} \right)^2 \right) \right) \\ + \frac{\partial}{\partial x^*} \left(\dot{\sigma}_i \left(\frac{1}{\gamma} T^* + \frac{\gamma - 1}{\gamma} \frac{1}{T^*} + \frac{1}{2} \left(\frac{\omega L_{ref}^3}{A_{ff} \sqrt{\gamma R T_{ref}}} \right)^2 \right. \right. \\ \left. \left. \times (\gamma - 1) \left(\frac{T^* \dot{\sigma}_i}{p^*} \right)^2 \right) \right) = \frac{h}{\rho \mu C_p} \frac{L_{ref}}{r_{hi}} \dot{\sigma}_i (T_w^* - T^*)\end{aligned}\quad (10)$$

From Eqs. (9) and (10), the DG $\omega L_{ref}^3 / A_{ff} \sqrt{\gamma R T_{ref}}$ can be pointed out.

Because the volume can also be expressed as $A_{ff} = V_i/L_i$, and $L_{ref}^3 = V_{sw}$, this DG becomes $\omega L_i / \sqrt{\gamma R T_{ref}} 1/DG_{Vi}$. The defined DG is finally:

$$DG_{Mi} = \frac{\omega L_i}{\sqrt{\gamma R T_{ref}}} \quad (11)$$

Note that DG_{Mi} can be seen as a modified Mach number as pointed out by Organ [17].

The friction factor f_F is related to the Reynolds number. From its general definition, $Re = \rho_i u_i r_{hi} / \mu$. Using the set of dimensionless variables, the Reynolds number can be written as:

$$Re = DG_{Mi}^2 \dot{\sigma}_i \frac{1}{DG_{Vi}} \frac{p_{mean} r_{hi}}{\omega \mu L_i} \quad (12)$$

Because DG_{Mi} and DG_{Vi} have been already defined, we define:

$$DG_{Ri} = \frac{p_{mean} r_{hi}}{\omega \mu L_i} \quad (13)$$

2.4. Thermal transfer

The thermal transfer coefficients h in Eq. (10) is related to the Stanton number ($St = h/\rho C_p u$).

Experimental correlations allow St to be expressed as a function of the Reynolds and the Prandtl ($Pr = C_p \mu / k$) numbers. For the heater and the cooler, turbulent flows are reported whereas laminar flow usually occurs in the regenerator. Hence, the correlations usually used for Stirling engines are chosen [13,19] as Eq. (14) for the heat exchangers and Eq. (15) for the regenerator.

$$St = Pr^{-2/3} \exp(0.337 - 0.812 \log(Re)) \quad (14)$$

$$St = Pr^{-2/3} \exp(-0.299 - 0.41 \log(Re)) \quad (15)$$

The ratio $h/\rho \mu C_p L_{ref}/r_{hi}$ appears in Eq. (10). As a consequence, using the previously defined DGs and the Eqs. (14) and (15), two new DGs are defined for the heater and cooler (Eq. (16)) and for the regenerator (Eq. (17)):

$$DG_{NTU_i} = 1.4 \left(\frac{C_p}{k} \right)^{-2/3} \left(\frac{L_i}{r_{hi}} \right)^{1.812} \left(\frac{p_{mean}}{\omega} \right)^{-0.812} \mu^{0.145} \quad (16)$$

$$DG_{NTU_r} = 0.74 \left(\frac{C_p}{k} \right)^{-2/3} \left(\frac{L_r}{r_{hr}} \right)^{1.412} \left(\frac{p_{mean}}{\omega} \right)^{-0.412} \mu^{-0.254} \quad (17)$$

2.5. Mechanical dynamics equations

Dynamic similarity is obtained from the dynamic equilibrium equations of the moving parts: Eq. (18) for the piston and Eq. (19) for the displacer.

$$m_p \ddot{x}_p = A_p p - c_p \dot{x}_p + A_{bp} p_{bp} \quad (18)$$

$$m_d \ddot{x}_d = A_{rod} p - c_d \dot{x}_d + A_d \sum_i \frac{1}{2r_{hi}} \frac{f_{Fi} \dot{m}_i |\dot{m}_i|}{\rho_i A_{ff}^2} L_i + A_{rod} p_{bd} \quad (19)$$

Using the dimensionless variables the Eqs. (18) and (19) become:

$$\ddot{x}_p^* = \frac{A_p p_{ref}}{\omega^2 m_p L_{ref}} p^* - \frac{c_p}{\omega m_p} \dot{x}_p^* + \frac{A_{bp} p_{ref}}{\omega^2 m_p L_{ref}} p_{bp}^* \quad (20)$$

$$\ddot{x}_d^* = \frac{A_{rod} p_{ref}}{\omega^2 m_d L_{ref}} p^* - \frac{c_d}{\omega m_d} \dot{x}_d^* + \frac{A_d}{m_d L_{ref}} \sum_i \frac{1}{2r_{hi}} \frac{f_{Fi} M V_{ref} \dot{\sigma}_i |\dot{\sigma}_i| T_i^*}{A_{Fi}^2} p^* L_i + \frac{A_{rod} p_{ref}}{\omega^2 m_d L_{ref}} p_{bd}^* \quad (21)$$

The natural dynamic of the FPSE is related to the first and the last terms of the right hand side of Eqs. (20) and (21). They account for the balance between pressure and inertia effects. The effects of the dissipative terms are supposed to be negligible for the operating frequency of the engine. The gas dissipative term has already been taken into account in DG_{Ri} .

Therefore, the specific dynamic of FPSE can be ensured during the scaling process with the following DGs:

$$DG_{Dp} = \frac{A_p p_{ref}}{\omega^2 m_p L_{ref}} \quad (22)$$

$$DG_{Dd} = \frac{A_{rod} p_{ref}}{\omega^2 m_d L_{ref}} \quad (23)$$

3. Method for preliminary design using the DGs

3.1. Comparison of the DGs for various Stirling engines

In order to support the proposed approach, the DGs can be evaluated for documented engines [17] namely: GPU-3, PD46, V160, MP1002CA, USSP40, 400HP. The FPSE RE1000 [20], CTPC [21], B10 [22] and DFPSE [23] have also been taken into account. Table 1 presents some of the relevant characteristics of these engines for scaling.

The ratio DG_{Mi}/DG_{Vi} appears in Eqs. (9) and (10). It is of importance in the underlying physics of the engine. Fig. 2a–c shows the relation between the DG_{Mi} and DG_{Vi} values for the regenerator, the cooler and the heater of various real Stirling engines. The different ratios DG_{Mi}/DG_{Vi} do not give the same values, though a global trend appears from Fig. 2a–c. This analysis backs up the proposed approach.

For the new proposed DGs for FPSE, Fig. 2d presents the ratio $\sqrt{A_p p_{mean}/m L_{ref}}$ as a function of the frequency which composes the dynamic groups DG_{Dp} and DG_{Dd} defined in Eqs. (22) and (23). Again, the correlation between these parameters proves that the proposed DGs are representative of the FPSE characteristics.

Similar conclusions can be drawn for the other DGs, but are not illustrated here for conciseness reason.

3.2. Use of the model

If we consider a given component (e.g. regenerator) the similitude would be achieved during the scaling if the DGs are kept

constant. Relationships between the parameters are obtained from the five DGs defined by Eqs. (1), (2), (11), (13), and (17). In logarithmic form, the 5 linear equations for 7 unknown variables are:

$$\begin{bmatrix} -1 & -1 & -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 2 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & -0.412 & 0.412 & -1.412 & 1.412 & 0 & 0 \end{bmatrix} \begin{bmatrix} \log(V_{sw}) \\ \log(p_{mean}) \\ \log(\omega) \\ \log(r_{hr}) \\ \log(L_r) \\ \log(n_r) \\ \log(P_m) \end{bmatrix} = \begin{bmatrix} \log(BeN) \\ \log(DG_{Vr}/4) \\ \log(DG_{Mr} \sqrt{\gamma R T_r}) \\ \log(DG_{Re} \mu) \\ \log(DG_{NTU_r}/0.74 \mu^{-0.254} (C_p/k)^{-2/3}) \end{bmatrix} \quad (24)$$

The swept volume is representative of the global dimensions of the engine and the used pressure, which should be as high as possible to increase the performance, can be typically given by technological constraints. Therefore, the displacement V_{sw} and the pressure p_{mean} are usually fixed. The remaining geometrical variables L_i , r_{hi} , n_i , the operating frequency and the estimated power can then be evaluated solving Eq. (24).

For the FPSE, as the operating frequency is given by the previous DGs. The values of the moving masses (m_p , m_d) and the areas (A_p , A_{rod}) have to be evaluated. The logarithmic expressions of the Eqs. (22) and (23) are:

$$\log(m_p) = -\log(DG_{Dp}) + \log(p_{mean}) - 1/3 \log(V_{sw}) - 2 \log(\omega) + \log(A_p) \quad (25)$$

$$\log(m_d) = -\log(DG_{Dd}) + \log(p_{mean}) - 1/3 \log(V_{sw}) - 2 \log(\omega) + \log(A_{rod}) \quad (26)$$

In order to solve these equations, one more assumption is needed. The piston and rod surfaces are supposed to scale as $V_{sw}^{2/3}$ (i.e. L_{ref}^2). By doing this, the mass values of the piston and the displacer are evaluated. This choice may be modified. For example, because of technical reason, these surfaces may not be scaled as $V_{sw}^{2/3}$ leading to other results for the masses. This would be the case if pistons are replaced by membranes.

3.3. Example of miniature geometries using similitude

The scale ε is defined from the derivative and the reference swept volumes as: $V_{sw}^{der} = V_{sw}^{ref} \cdot \varepsilon^3$.

Table 1
Characteristics of the documented Stirling engines.

Engine	MP1002CA	GPU-3	USSP40	Philips 400HP	PD46	V160	RE1000	CTPC	B10	DFPSE
Type	Kinematic						Free piston			
Characteristics DGs	BeN; DG_{Vi} ; DG_{Mi} ; DG_{Ri} ; DG_{NTU_i}						BeN; DG_{Vi} ; DG_{Mi} ; DG_{Ri} ; DG_{NTU_i} ; DG_{Di}			
Gas	Air	H ₂	H ₂	He	He	He	He	He	He	He
P_m (kW)	0.25	8.95	7.5	291	3	—	1	14.5	—	—
p_{ref} (bar)	14	70	150	110	102	150	70	150	1	10
f_{ref} (Hz)	25	60	33	47	314	157	30	70	13	33
V_{sw} (cm ³)	61.1	118	134	15 300	77.5	225	114.2	407.6	22	384
L_{ref} (mm)	39.4	48.8	51.1	248.2	42.6	60.8	48.5	74.1	28	72.7
m_p (kg) if relevant	—	—	—	—	—	—	6.2	13.2	0.5	6.2
m_d (kg) if relevant	—	—	—	—	—	—	0.42	2.17	0.08	0.42

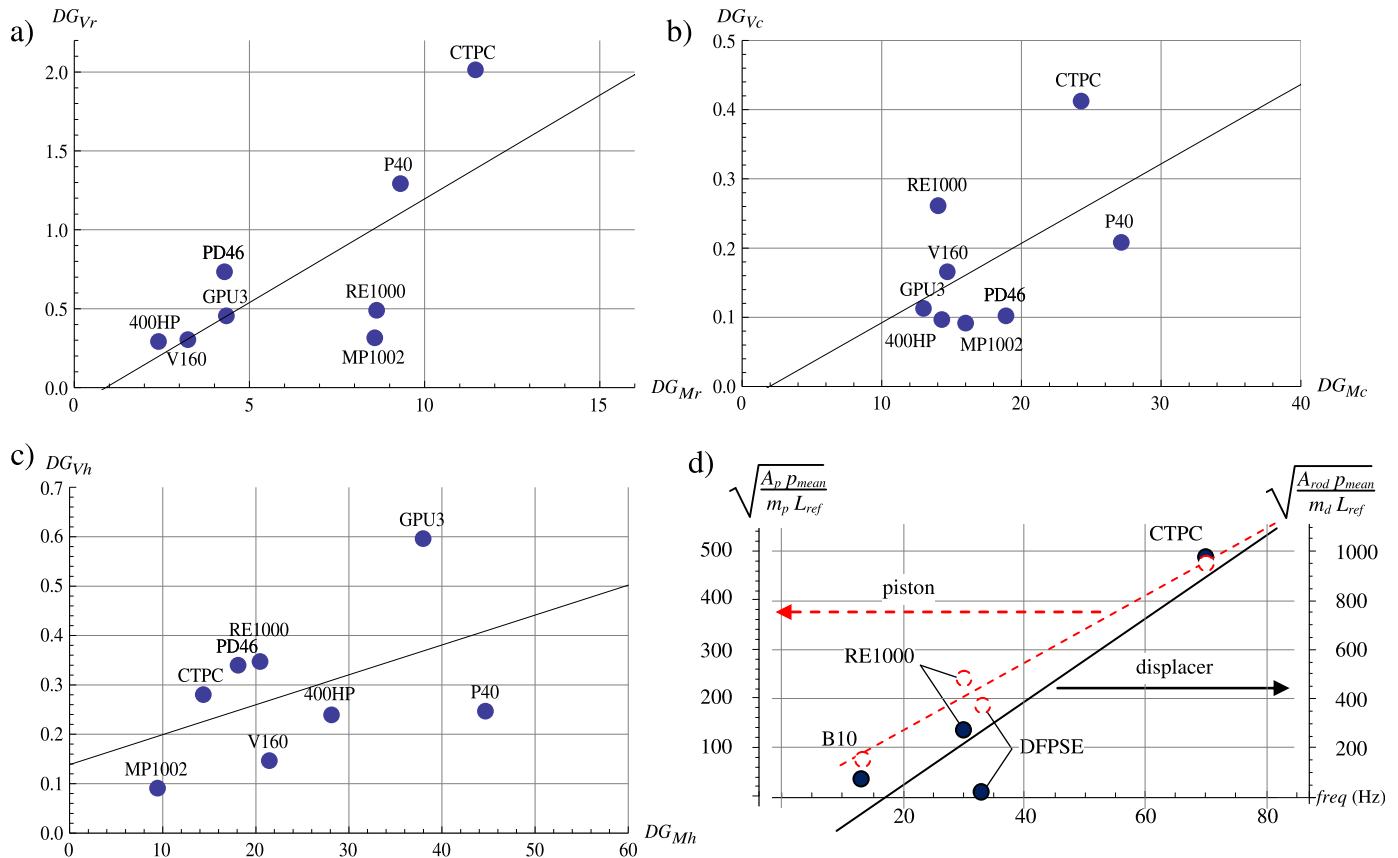


Fig. 2. Energetic correlation for the regenerator of various Stirling engines. a) Regenerator, b) cooler, c) heater, d) dynamic correlation for FPSE.

For pressures varying from 10 to 70 bars, and displacements from 5 to 0.1 mm³ (ϵ range [0.01, 0.04]), test cases are evaluated from the fully documented FPSE RE1000 [20] for the same temperatures ($T_h \approx 550^\circ\text{C}$, $T_c \approx 60^\circ\text{C}$). The scaling strategy consists in setting the scale coefficient (i.e. the displacement V_{sw}) and the mean pressure (p_{mean}) and evaluating the operating frequency and the geometric parameters for all the heat components (heater, cooler, regenerator) and the masses.

From the detailed engine parameters it is possible to obtain a preliminary design of a micro-Stirling defining the lengths, the

hydraulic radii, the numbers of channels and the masses of the engine. Results are given in Table 2.

The evolutions of the length or hydraulic radius are shown to be not proportional to the scale (e.g. comparing cases #1 and #2 in Table 2). The regenerator lengths range from 6 to 14 mm for pressurized engine, which is consistent with the literature [11,12].

As a crucial component, the evolution of the geometry of the regenerator is underlined in Fig. 3. Solving Eq. (24) for the DGs values of the RE1000, we can estimate that for the regenerator, the hydraulic radius r_{hr} scales as $p_{mean}^{-0.58} V_{sw}^{0.14}$ (or $p_{mean}^{-0.58} \epsilon^{0.42}$

Table 2

Geometric and operating parameters for four derived micro-engines. The bold figures are the chosen parameters from which the similitude process is performed.

	Reference (RE1000 [20])	Scaling case #1	Scaling case #2	Scaling case #3	Scaling case #4
Swept volume V_{sw} [mm ³]	69 000	5	0.5	0.5	0.1
Scale ϵ	1	1/24 (≈ 0.04)	1/52 (≈ 0.02)	1/52 (≈ 0.02)	1/88 (≈ 0.01)
Op. pressure p_{mean} [bar]	70	70	70	10	10
Op. frequency $f = \omega/2\pi$ [Hz]	191	191	300	134	183
Displacer $\varnothing D_d$ [mm]	57.2	2.3	1.1	1.1	0.6
Cooler					
Length L_c [mm]	79.2	10.2	5.24	12.4	8.8
Hyd. radius r_{hc} [μm]	223	72	55	193	160
Heater					
Length L_h [mm]	183.4	23.7	16.7	20.4	14.4
Hyd. radius r_{hh} [μm]	590	191	158	720	595
Regenerator					
Length L_r [mm]	64.4	10	6.4	14.4	10.5
Hyd. radius r_{hr} [μm]	70	19	14	43	34
Wire $\varnothing d_w$ [μm]	89	24	17	54	43
Porosity η [%]	75.9	75.9	75.9	75.9	75.9
Max. Knudsen number (regenerator)	$8.63 \cdot 10^{-5}$	$3.2 \cdot 10^{-4}$	$4.4 \cdot 10^{-4}$	$9.9 \cdot 10^{-4}$	$1.23 \cdot 10^{-3}$

using the scale ϵ) and the length L_r as $p_{\text{mean}}^{-0.41} V_{\text{sw}}^{0.19}$ or $p_{\text{mean}}^{-0.41} \epsilon^{0.57}$. The test cases #1–4 are presented as black circles.

The black line is the usual proportional scaling law for 70 bars (the length scales as ϵ). According to the similitude approach, the proposed dimensions theoretically ensure a similar effectiveness compared to the reference engine. Fig. 3 clearly shows that proportional scaling would not lead to efficient regenerator. Moreover, moderate pressures tend to increase the dimensions.

3.4. Model assumption validity

The proposed approach relies on the consistency of the continuous flow theory. The minimal dimensions of the engine that can be designed using the proposed FOM are then defined here as the lowest swept volume associated to the rarefaction effect limit.

It can be assessed evaluating the Knudsen number Kn . The minimal equivalent hydraulic radius is the reference length for the regenerator component. As an example, it has been given for the four test cases in Table 2. If the mean free path of the molecule is far smaller than the characteristic length of the considered problem ($Kn < 10^{-3}$), then the continuum theory can be applied and so the proposed model. The Knudsen numbers are evaluated for the regenerator and are given in Fig. 4.

It appears that a size reduction of about $\epsilon = 0.01$ ($V_{\text{sw}} = 0.1 \text{ mm}^3$) for a minimal operating pressure of 20 bars, represents the validity limit for the model. Under these dimensions, usually neglected phenomena may have to be taken into account [24].

Some theoretical works studied the effect of quantum degeneracy of the performances of Stirling cooler. Quantum degeneracy of gas appears at very low temperatures or for dimensions under micrometer scale. Although, some results suggest that operating a Stirling cycle machines at nanoscale is possible [25,26], such studies are beyond the scope of this paper.

4. Scaling laws for FPSE

Scaling analysis for Stirling machines has been proposed in Refs. [10–12]. They are based on the effects of a geometrical scaling on the various relevant phenomena for the Stirling machine as described in Table 3. In these studies, any length scale as ϵ , area as ϵ^2 and volume as ϵ^3 . The scaling laws derived from the expression of the different effects and losses are called “CSL” (classical scaling laws) in the following.

Using the proposed FOM which takes into account the coupling in Stirling engine, NSL (new scaling laws) are defined. They stem on

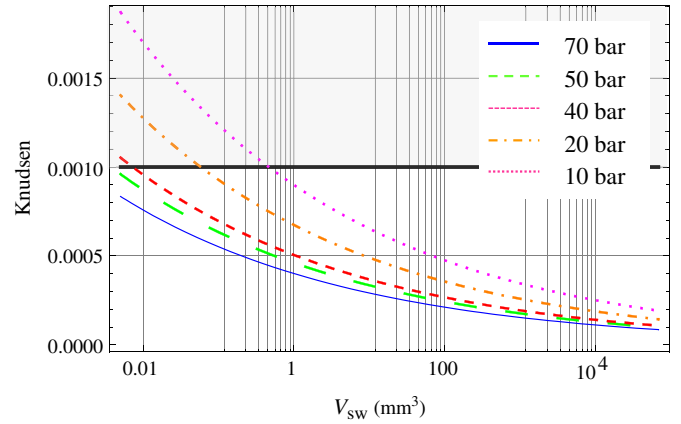


Fig. 4. Evaluation of the Knudsen number associated to the regenerator.

the introduction of the solution of Eqs. (24)–(26) in the expression of the losses and power.

In this study, the effect of scaling on the frequency, the regenerator losses and the output power are taken into account. Additionally, the effect of a gap between the displacer and the housing is considered. This is thought to be especially relevant because of the problem of sealing and manufacturing tolerance at small scale. The output power and losses are also proposed as specific quantities by dividing their expression by the representative volume V_{sw} .

4.1. Operating frequency and specific power

The operating frequency is related to the square root of the stiffness to the mass ratio. Whether associated to the elastic strain of material (mechanical spring) or the adiabatic gas-spring, the stiffness scales as ϵ [10]. Because, for CSL, the mass scales as ϵ^3 [10], the frequency law is ϵ^{-1} . This evolution is the black curve plotted in Fig. 5a.

Using the FOM, the natural frequency ω is given by the solution of the set of Eq. (24) when p_{mean} and ϵ ($\epsilon = V_{\text{sw}}^{1/3}$) are fixed. ω is a then function of the DGs, the gas characteristics, p_{mean} and ϵ . Then, the new scaling law for the frequency is calculated as: $\omega \propto p_{\text{mean}}^{0.41} \epsilon^{-0.58}$.

The evolutions of the frequency according to the NSL are plotted in Fig. 5 for different pressures. Compared to the CSL, the operating frequency which theoretically ensures the same level of

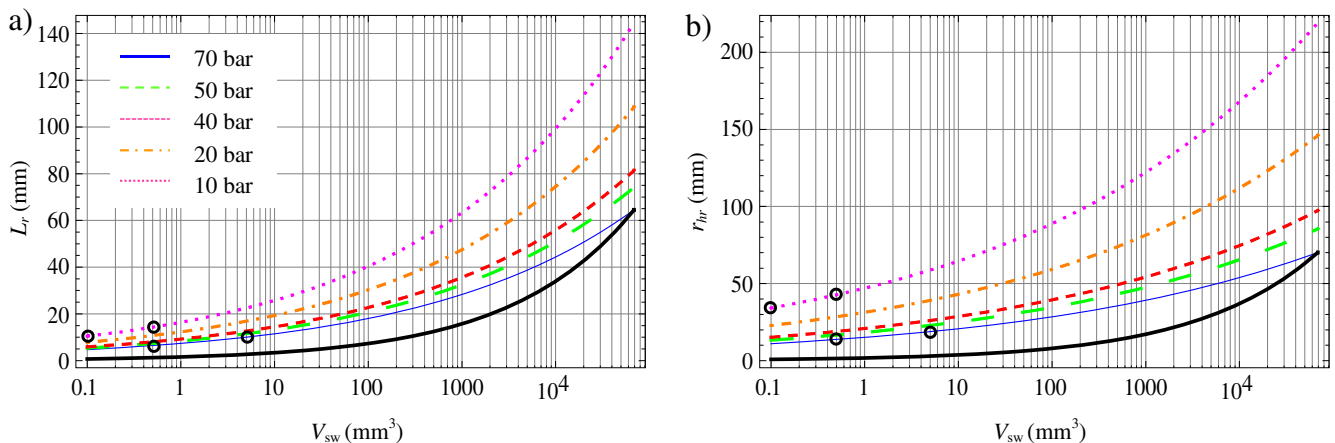


Fig. 3. a) Length of the regenerator as a function of the displacement V_{sw} , b) hydraulic radius for the regenerator as a function of the displacement V_{sw} .

Table 3
Scale effects on the various relevant physical phenomena for Stirling machines.

Phenomena	Expression	Units	Classical scaling law	Scaling law from FOM	Comments
<i>Dynamic parameters</i>					
Natural frequency	$\omega \equiv \sqrt{\frac{K}{M}}$	rad s ⁻¹	ϵ^{-1} [10]	$p_{\text{mean}}^{0.41} \epsilon^{-0.58}$	
Mass	$m_p \equiv \rho V_p$	kg	ϵ^3 [10]	$p_{\text{mean}}^{0.18} \epsilon^{2.16}$	For piston or displacer
<i>Regenerator</i>					
Gas flow	$\dot{m} \equiv \omega \frac{p_{\text{mean}} V_{\text{ref}}}{RT_{\text{ref}}} [11]$	m ³ s ⁻¹	$p_{\text{mean}} \epsilon^2$	$p_{\text{mean}}^{1.41} \epsilon^{2.42}$	Constant T_{ref}
Pressure drop	$\Delta p \equiv \frac{f_f \dot{m}^2 L_r}{\rho A_r^2 r_h} [12]$	Pa	p_{mean}^2	$p_{\text{mean}}^{2.16} \epsilon^{0.18}$	Constant f_f
Regenerator reheat thermal loss	$Q_r \equiv (1 - \eta_r) \dot{m} C_p \Delta T [12]$	W	$p_{\text{mean}} \epsilon^2$	$p_{\text{mean}}^{1.41} \epsilon^{2.42}$	Constant ΔT η_r is the regenerator efficiency
Conduction	$Q_{\text{cond}} \equiv \frac{k A_r}{L_r} \Delta T [10-12]$	W	ϵ	$p_{\text{mean}}^{0.83} \epsilon^{1.83}$	Cst ΔT For the regenerator characteristics
<i>Gap</i>					
Gas flow	$\dot{m} \equiv \frac{\rho}{12\mu} \frac{D_{\text{gap}}^3}{L_r} \Delta p [30]$	m ³ s ⁻¹	$p_{\text{mean}}^2 \epsilon$	$p_{\text{mean}}^{2.57} \epsilon^{1.59}$	Gap leakage assuming that the gap scales as $\epsilon^{1/3}$
Leakage reheat thermal loss	$Q_{\text{gap}} \equiv \dot{m} C_p \Delta T$	W	$p_{\text{mean}}^2 \epsilon$	$p_{\text{mean}}^{2.57} \epsilon^{1.59}$	For gap leakage, with cst ΔT
<i>Gas-spring</i>					
Hysteresis gas loss	$Q_{\text{hys}} \equiv \sqrt{\frac{1}{32}} \omega \gamma^3 (\gamma - 1) T_w p_{\text{mean}} k \left(\frac{V_{\text{sw}}}{V_b} \right)^2 A_w [14]$	W	$p_{\text{mean}} \epsilon^{1.5}$	$p_{\text{mean}}^{0.71} \epsilon^{1.71}$	
<i>Power</i>					
Output power	P_m	W	$p_{\text{mean}} \epsilon^2$	$p_{\text{mean}}^{1.41} \epsilon^{2.41}$	From BeN
<i>Specific quantities</i>					
Specific output power	$P_m^* \equiv P_m / V_{\text{sw}}$	W m ⁻³	$p_{\text{mean}} \epsilon^{-1}$	$p_{\text{mean}}^{1.41} \epsilon^{-0.59}$	
Hysteresis gas loss	$Q_{\text{hys}}^* \equiv Q_{\text{hys}} / V_{\text{sw}}$	W m ⁻³	$p_{\text{mean}} \epsilon^{-1.5}$	$p_{\text{mean}}^{0.71} \epsilon^{-1.29}$	
Conduction loss	$Q_{\text{cond}}^* \equiv Q_{\text{cond}} / V_{\text{sw}}$	W m ⁻³	ϵ^{-2}	$p_{\text{mean}}^{0.83} \epsilon^{-1.17}$	Constant ΔT
Regenerator reheat thermal loss	$Q_r^* \equiv (1 - \eta_r) \dot{m} C_p \Delta T / V_{\text{sw}}$	W m ⁻³	$p_{\text{mean}} \epsilon^{-1}$	$p_{\text{mean}}^{1.41} \epsilon^{-0.58}$	With constant ΔT and η_r
Leakage reheat thermal loss	$Q_{\text{gap}}^* \equiv \dot{m} C_p \Delta T / V_{\text{sw}}$	W m ⁻³	$p_{\text{mean}} \epsilon^{-2}$	$p_{\text{mean}}^{2.57} \epsilon^{-1.41}$	For gap leakage, with cst Δp
<i>Viscous dissipations</i>					
	$P_{\text{vis}}^* \equiv \frac{\dot{m}}{\rho} \Delta p / V_{\text{sw}}$	W m ⁻³	$p_{\text{mean}}^3 \epsilon^{-1}$	$p_{\text{mean}}^{3.57} \epsilon^{-0.4}$	For the regenerator flow, with scaled Δp
	$P_{\text{visgap}}^* \equiv \frac{\dot{m}}{\rho} \Delta p / V_{\text{sw}}$	W m ⁻³	$p_{\text{mean}}^4 \epsilon^{-2}$	$p_{\text{mean}}^{4.73} \epsilon^{-1.23}$	For the gap leakage flow, with scaled Δp

performances for the exchangers (*i.e.* heat transfer coefficient and friction factor) has to be much lower. Therefore, the mass of the pistons will have to be chosen properly which is the point of DG_{Dp} and DG_{Dd} and will be discussed in the following.

The operating frequency of the FPSE partly relies on the moving masses. Fig. 6 shows the evolution of the displacer mass with respect to the scale ϵ . The thick black line stands for the CSL (ϵ^3 as can be read in Table 3). Using the dynamical results from similitude, the mass scales as $p_{\text{mean}}^{0.18} \epsilon^{2.14}$. For small scale variation, the evolutions are close (Fig. 6a). However, for the smallest engines, drastic differences can be seen in Fig. 6b. Assuming that the stiffness effect is associated to gas springs, the proposed new scaling of the displacer mass ensures the operating frequency to be optimal. It

can also be inferred that the relative mass with respect to the engine size will be more important in a micro-Stirling than in its macro-scale counterpart.

The output power is related to the operating frequency. Fig. 7 presents the CSL (black curve) and the NSL for the specific output power. The power density increase when miniaturizing is predicted by both the scaling laws. However, on the one hand, the frequency has to be tuned; on the other hand the geometrical parameters have to vary properly to maintain the heat exchange properties. This is ensured by the proposed laws whereas the classical law would lead to too high frequency (Fig. 5) and too small geometry (Fig. 3).

Therefore, the proposed evolution of the power density is thought to be more relevant. Still, cubic millimeter displacement engine ($\epsilon \approx 0.04$) can present up to 7 times more specific power than macro-scale engine. The role of the pressure is presented in Fig. 7. The power density is shown to be not proportional to the pressure which is a new result. Again, the maximal attainable pressure is to be sought to increase the specific power of the engine.

4.2. Losses

The FOM theoretically ensures the same behavior for the heat exchangers and the regenerator. Then, the effectiveness of the regenerator, the heat exchange coefficient and the friction factor are theoretically constant. However, the losses phenomena have to be analyzed to anticipate the engine efficiency. Using a previously developed model for FPSE [9], the losses have been evaluated for the RE1000 and are used here to obtain their initial distributions at full scale ($\epsilon = 1$, subscript FS in the following). Then, their evolutions with scaling can be anticipated using the NSL.

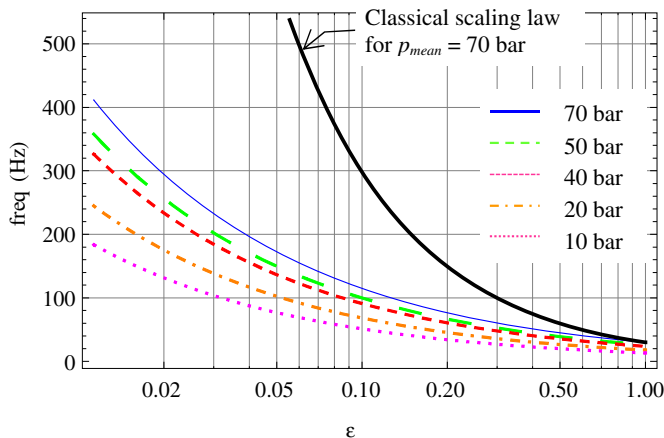


Fig. 5. Operating frequency for pressure from 10 to 70 bars.

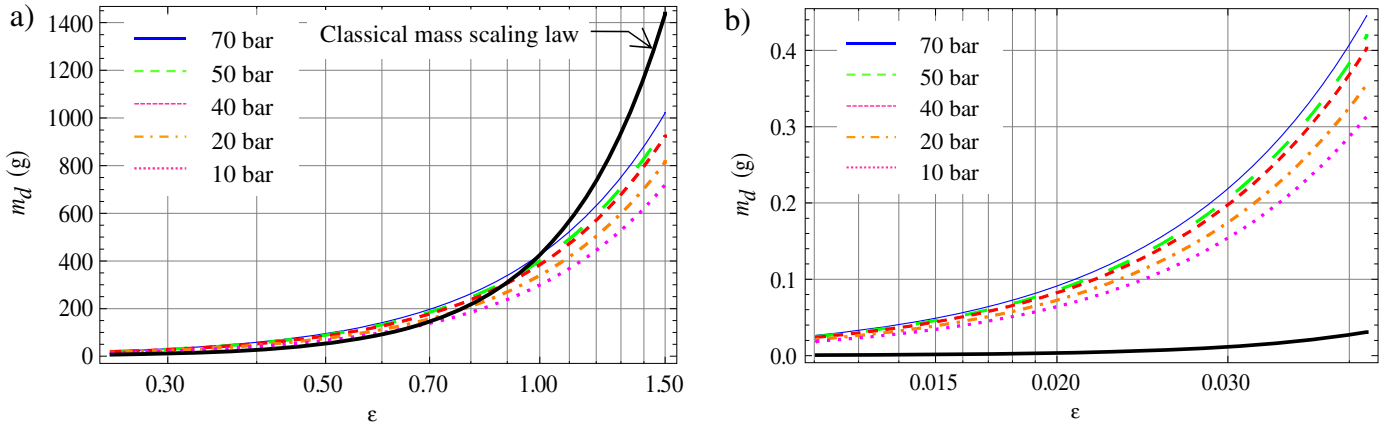


Fig. 6. a) Displacer mass as a function of the scale for ϵ range [1/4, 3/2], b) for ϵ range [1/88, 1/24].

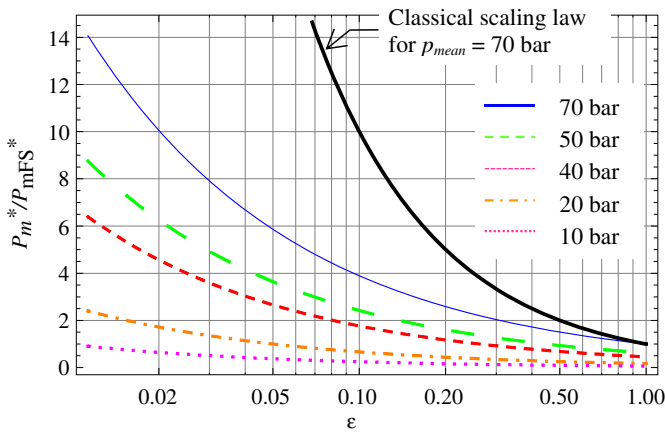


Fig. 7. Power density evolution for pressure from 10 to 70 bars.

4.2.1. Conduction losses

The CSL for the heat conduction across the regenerator suggests that the associated specific heat loss scales as ϵ^{-2} as can be read in Table 3 (black curve in Fig. 8a). Following the similitude, it can be inferred that the same loss scales as $p_{mean}^{0.83} \epsilon^{-1.17}$. The conduction loss increase is smaller than for the CSL because the regenerator length scales as $\epsilon^{0.57}$, (see Figs. 3 and 8a). Additionally, this new

result underlines the indirect effect of the pressure which could be used to reduce the conduction loss.

Fig. 8b plots the evolution of the percentage of the specific conduction losses relative to the total losses. It can be seen that the conduction can account for up to 60% of the total losses for mm³ displacement engine ($\epsilon \approx 0.04$) and is superseded by other losses for smaller engines. Note that the conduction loss influence is enhanced by low pressure which means that other losses may drastically lessen with pressure.

4.2.2. Regenerator losses

Regenerator losses are: the reheat thermal losses associated to its effectiveness and the pressure drop. Its effectiveness η_r is theoretically constant since it can be expressed as a function of the NTU [12] which is taken into account in $DG_{NTU,r}$.

The increase of the thermal loss when miniaturizing (Fig. 9a) appears to be moderate in comparison with the evolution of the conduction loss. Moreover, from the full scale value, the weight of this loss is only 0.7% of the total loss and this value decreases as other losses prevail when miniaturizing (Fig. 9b).

The NSL for the pressure drop loss is obtained assuming a constant friction factor coefficient which is in accordance with the constant $DG_{R,r}$. The increase of this loss appears to be much less than for the CSL (Fig. 9c). Moreover, reducing the pressure can drastically diminish this loss. Using a pressure of 10 bars makes the contribution of the pressure drop loss in the regenerator less than 1% of the total losses (Fig. 9d).

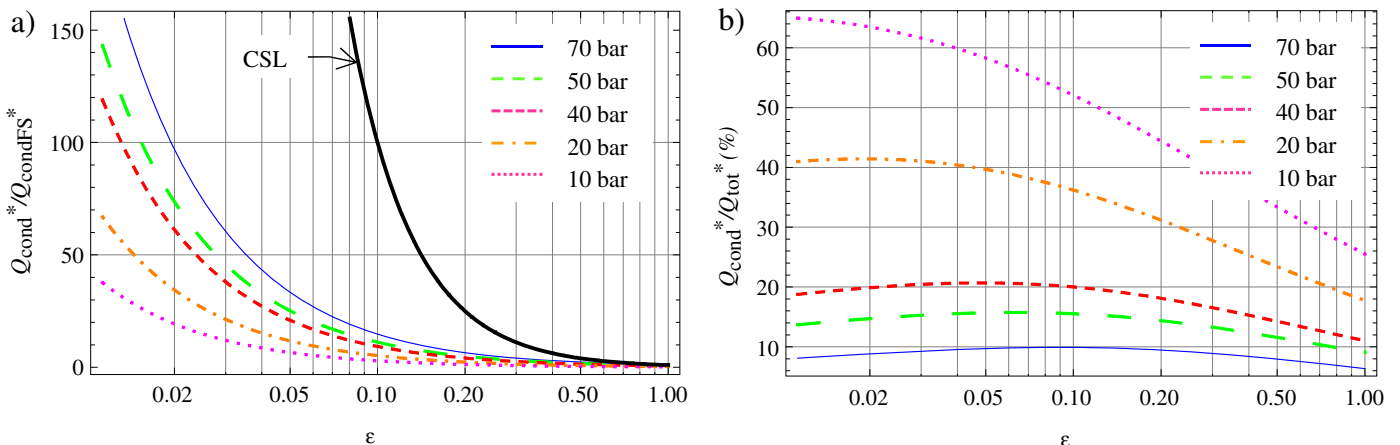


Fig. 8. a) Ratio of the specific conduction loss evolution to the full scale specific conduction, b) percentage of the specific conduction losses relative to the total losses.

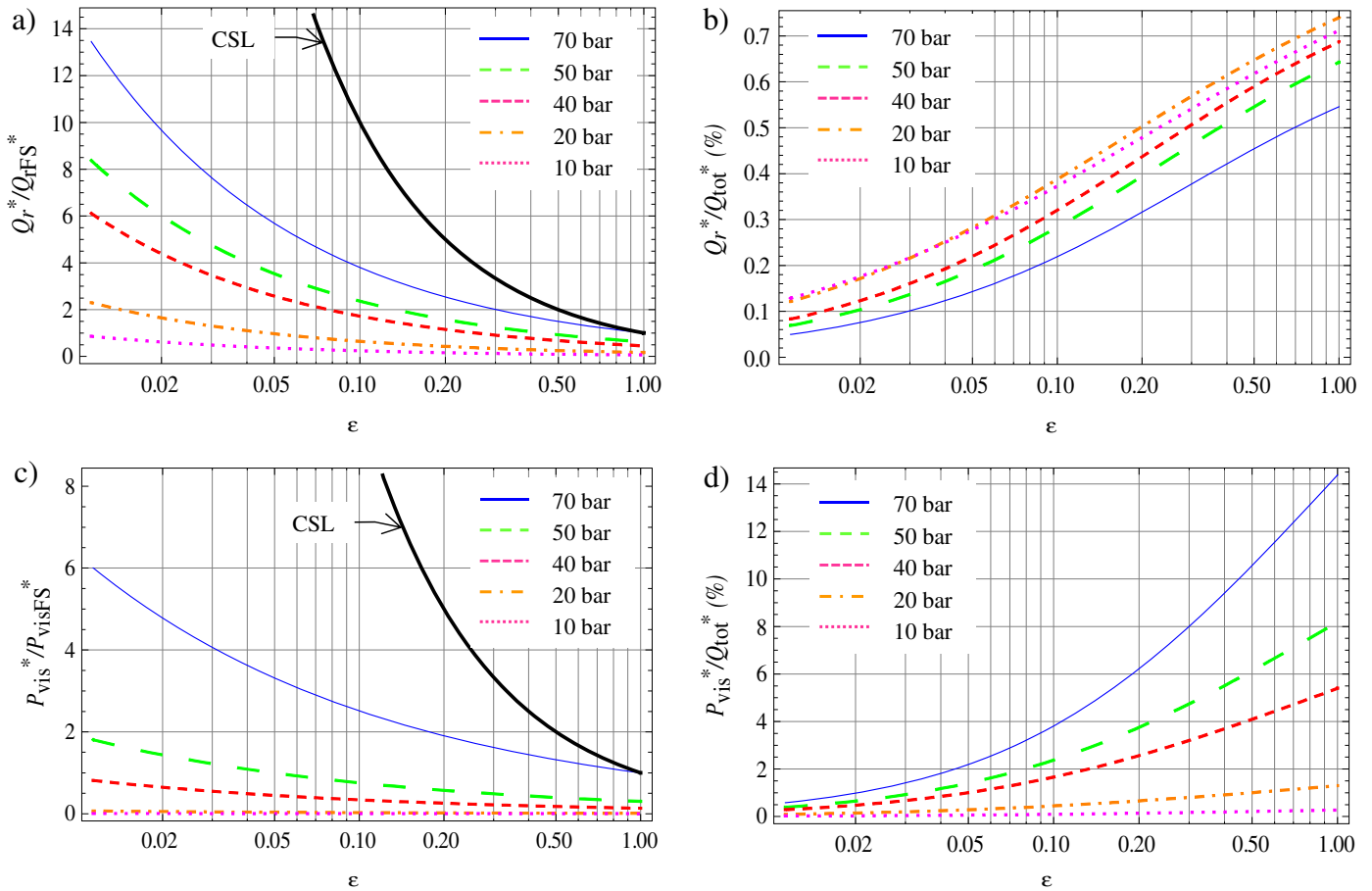


Fig. 9. a) Relative specific regenerator reheat thermal loss evolution, b) evolution of the ratio of the reheat thermal loss to the total losses, c) relative specific pressure drop loss evolution, d) evolution of the ratio of the specific pressure drop loss to the total losses.

4.2.3. Gas spring hysteresis

Another damping effect is the gas spring hysteresis. For FPSE it can also modify the phase angle and the amplitudes of the piston and displacer, which affects both power and efficiency. From Fig. 10a, it can be seen that the miniaturization greatly increases the hysteresis loss. However, in the case of the RE1000, this loss is small compared to the total of losses as can be seen in Fig. 10b for $\epsilon = 1$. As long as the RE1000 is used as a

reference engine for preliminary design, the gas-spring loss is not critical.

4.2.4. Effect of the gap gas leakage

From a technological point of view, materials and fabrication constraints are challenging when designing micro-engines. From the theoretical prospective studies, friction losses and leakage through the cylinder–piston gap was underlined as one of the main

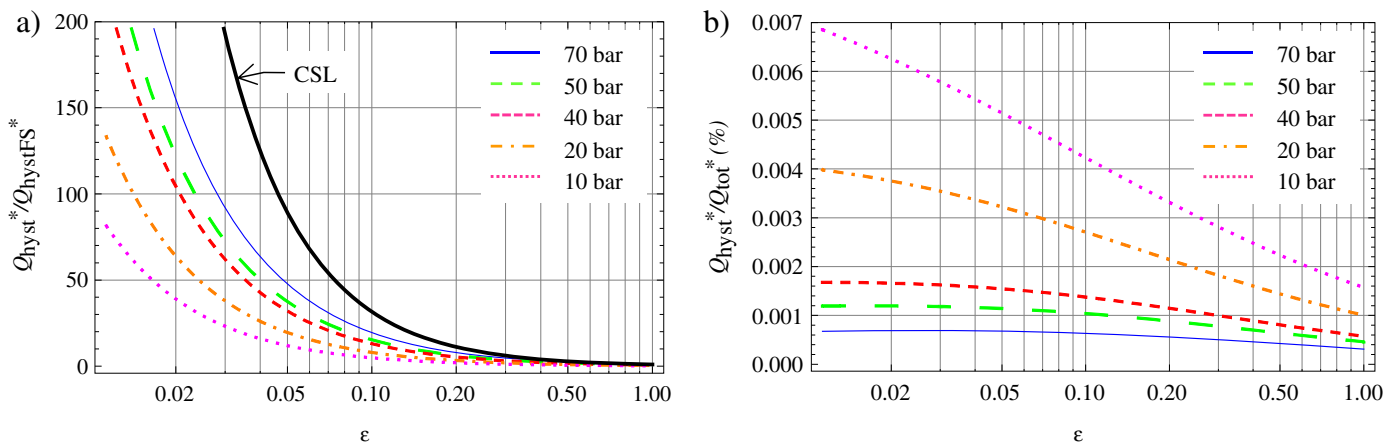


Fig. 10. a) Specific gas spring hysteresis losses evolution, b) evolution of the ratio of the specific hysteresis losses to the total losses.

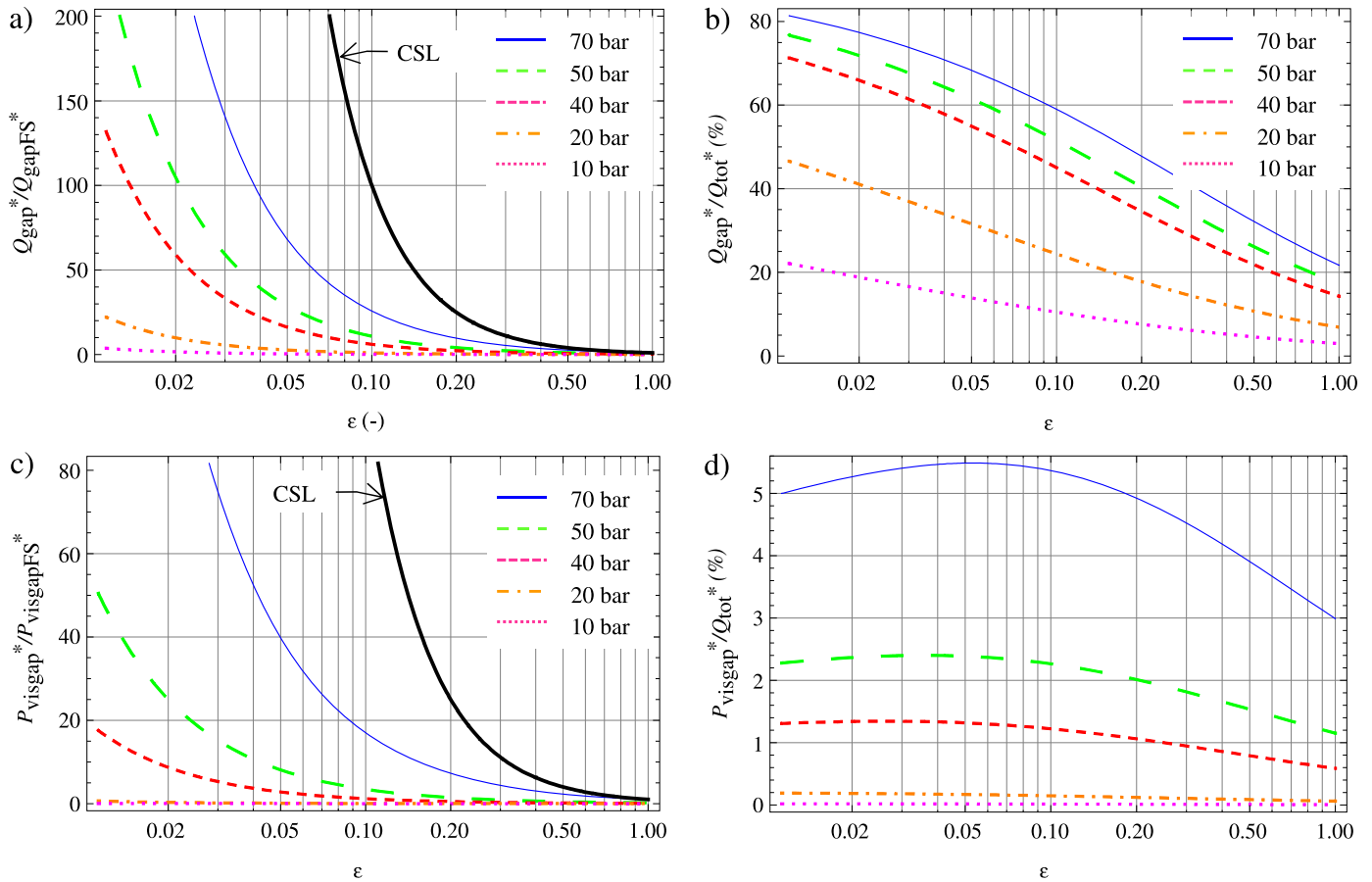


Fig. 11. a) Relative specific gap reheat thermal loss evolution, b) evolution of the ratio of the reheat thermal loss to the total losses, c) relative specific pressure drop loss evolution for the gap, d) evolution of the ratio of the specific pressure drop loss to the total losses.

limitations [1,2,27,28]. Gas leakage through the displacer-casing clearance induces reheat losses due to the flow of cold gas into the hot expansion chamber which impact the efficiency of the engine. In classical scale analyses, the geometrical gap is supposed to scale as ε . We choose here to highlight the technological point of view. We assume that no sealing component is used. From the engineering tolerance theory [29], a classical IT5 quality fitting is supposed to be adopted and it can be inferred that it scales as $\varepsilon^{1/3}$.

Due to the gap between the displacer and the casing a gas flow occurs. Its evaluation can be obtained using the hydrodynamic lubrication theory [30]. In addition to the pressure drop, a reheat thermal loss is considered. The gas flow through the gap strongly increases when miniaturizing and both the reheat and pressure losses increase (Fig. 11a and c).

The reheat thermal loss has the most important detrimental effect as can be seen in Fig. 11b. If the pressure is decreased,

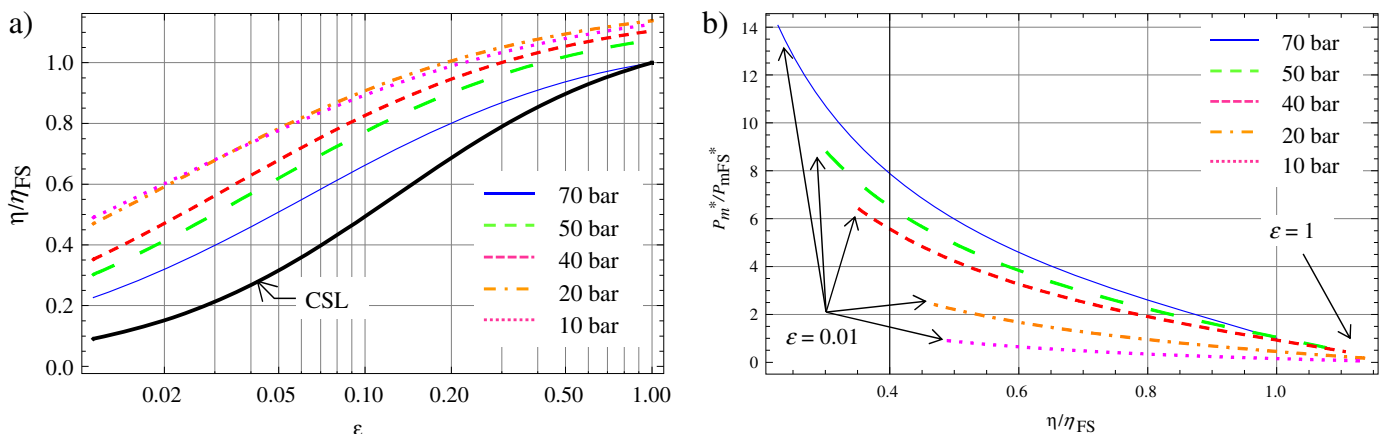


Fig. 12. a) Relative efficiency, b) relative specific power as a function of the relative efficiency for a scale $\varepsilon \in [0.01, 1]$.

the effect of the gap weakens. In this case, the conduction loss becomes the major source of loss (see Fig. 8b for $p_{\text{mean}} = 10$ bars).

4.3. Efficiency

The efficiency of the micro-Stirling engine is estimated here as the output mechanical power P_m divided by the total input thermal power following Eq. (27):

$$\eta = \frac{P_m}{(P_m + P_{\text{vis}} + P_{\text{visgap}}) \frac{1}{\eta_C} + Q_{\text{cond}} + Q_{\text{gap}} + Q_{\text{hyst}}} \quad (27)$$

in which η_C is the Carnot efficiency.

The relative efficiency with respect to the scale ε is shown in Fig. 12a. Using CSL would lead to small efficiency (black curve in Fig. 12a). The study of the various losses showed that a moderate pressure tends to reduce the increase for all the losses and a better efficiency can be obtained.

Fig. 12b presents the specific power and efficiency evolution with respect to ε . Though, the efficiency is improved with lower pressure, the effect is much more important on the specific power and reduces it drastically. As an example for $p_{\text{mean}} = 40$ bars (dotted red curve in the web version), the efficiency is 30% lower than for $p_{\text{mean}} = 10$ bars but the power is six times greater.

As a conclusion, some hints for designing micro-Stirling engine can be deduced from this analysis. First the leakage between the displacer and the housing induces the main source of loss. Sliding piston should be avoided. Low pressures reduce this loss but have a great impact on the power. The conduction loss appears to be the second most important source of loss. Therefore, if membranes are used instead of piston, thermal conduction through the membrane thickness would result in much higher conduction losses. It appears that pressurization is more important in micro-Stirling than in its macro-scale counterpart. It may be an explanation for the fails of the few experimental micro-Stirling design attempts using atmospheric pressure [31,32].

From the dynamical point of view, it was demonstrated that the inertia effect has to be stronger for miniature engines than for their macro-scale counterparts. New membrane structures are then to be designed to match the thermal isolation and the inertia requirements, such as proposed in Ref. [33].

5. Conclusions

The effects of size reduction on performances have been studied for the FPSE, which is the best suited Stirling architecture for miniaturization. From the similitude approach a simple first order model has been proposed. It allows a preliminary design to be obtained from a full scale engine. The geometrical parameters (length, hydraulic radius, number of channel) for the heat exchangers and the regenerator as well as the operating frequency and the masses of the pistons are obtained.

Moreover, the evolution laws of these parameters allow new scaling laws for the power and the major sources of losses to be defined. Conclusions from these laws suggest that the specific power of a Stirling engine can be significantly increased with size reduction and using high pressure. However, the increase is less than predicted by the classical geometrical scaling law and the new results are thought to be more realistic. Besides, the role of the pressure is underlined and clearly stated by the proposed scaling laws. Matching the optimal operating frequency allows to maintain high effectiveness for the heat exchangers and the regenerator. It requires that the moving masses are properly designed. The relative inertia with respect to the engine dimensions increases as the size decreases which lead to larger moving components.

The detrimental effect of gap leakage on the engine efficiency has been underlined. Because the gap induces most of the losses, the sliding piston architecture appears to be not suitable for micro-engine. The use of membrane instead of the piston and the displacer could be a solution to this issue. However, the membrane would have to ensure high thermal isolation and to match the frequency requirement which means high inertia effect.

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